

The nature of massive neutrinos and Multiple Mechanisms in Neutrinoless Double Beta Decay ^a

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Determining the nature —Dirac or Majorana— of massive neutrinos is one of the most pressing and challenging problems in the field of neutrino physics. We discuss how one can possibly extract information on the couplings, if any, which might be involved in $(\beta\beta)_{0\nu}$ -decay using a multi-isotope approach. We investigate as well the potential of combining data on the half-lives of nuclides which largely different Nuclear Matrix Elements such as ^{136}Xe and of one or more of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , for discriminating between different pairs of non-interfering or interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay. The case studies do not extend to the evaluation of the theoretical uncertainties of the results, due to the nuclear matrix elements calculations and other causes.

I. ARE NEUTRINOS MAJORANA PARTICLES?

All neutrino oscillation data can be described within the reference 3-flavour neutrino mixing scheme, with 3 light neutrinos ν_j , having masses $m_j \lesssim 1$ eV, $j = 1, 2, 3$ (see, e.g., [1]). These data allowed to determine, with a relatively high precision (see [2, 3] for recent global fit analysis), the parameters which drive the observed solar, atmospheric, reactor and accelerator flavour neutrino oscillations — the three neutrino mixing angles θ_{12} , θ_{23} and θ_{13} of the standard parametrisation of the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) neutrino mixing matrix U and the two neutrino mass squared differences Δm_{21}^2 and Δm_{31}^2 (or Δm_{32}^2).

Despite the great successes of the last decades, we still have no clue about the nature of massive neutrinos, which could be Dirac or Majorana.

The Majorana nature of neutrinos manifests itself in the existence of processes in which the total lepton charge L changes by two units: $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$, $\mu^- + (A, Z) \rightarrow \mu^+ + (A, Z - 2)$, etc. Extensive studies have shown that the only feasible experiments having the potential of establishing that the neutrinos are Majorana particles are at present the experiments searching for neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay (see e.g. for a review [4]):

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-. \quad (1)$$

Under the assumptions of 3- ν mixing, with neutrinos ν_j being Majorana particles, and of $(\beta\beta)_{0\nu}$ -decay generated only by the $(V - A)$ charged current weak interaction via the exchange of the three Majorana neutrinos ν_j , the $(\beta\beta)_{0\nu}$ -decay amplitude of interest has the form (see, e.g. [5, 6]):

$$\mathcal{A}(\beta\beta)_{0\nu} = \langle m \rangle M(A, Z), \quad (2)$$

where $M(A, Z)$ is the nuclear matrix element (NME) of the decay in eq. (1) which does not depend on the neutrino mass and mixing parameters, and

$$|\langle m \rangle| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}|, \quad (3)$$

is the $(\beta\beta)_{0\nu}$ -decay effective Majorana mass. In eq. (3), U_{ej} , $j = 1, 2, 3$, are the elements of the first row of the PMNS matrix U , and α_{21} and α_{31} are the two Majorana CP violation (CPV) phases contained in U . In the standard parametrization of the PMNS matrix U (see, e.g., [1]), the phase

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α_{31} in eq. (3) must be replaced by $(\alpha_{31} - 2\delta)$, δ being the Dirac CPV phase present in U , and $|U_{e1}| = \cos \theta_{12} \cos \theta_{13}$, $|U_{e2}| = \sin \theta_{12} \cos \theta_{13}$, $|U_{e3}| = \sin \theta_{13}$.

We recall that the predictions for $|\langle m \rangle|$ depend on the type of the neutrino mass spectrum [6–8]. As is well known, depending on the sign of $\Delta m_{31(32)}^2$, which cannot be determined from the presently available neutrino oscillation data, two types of neutrino mass spectrum are possible:

- i) *normal ordering (NO)*: $m_1 < m_2 < m_3$, $\Delta m_{31}^2 > 0$, $\Delta m_{21}^2 > 0$, $m_{2(3)} = (m_1^2 + \Delta m_{21(31)}^2)^{1/2}$;
- ii) *inverted ordering (IO)*: $m_3 < m_1 < m_2$, $\Delta m_{32}^2 < 0$, $\Delta m_{21}^2 > 0$, $m_2 = (m_3^2 + \Delta m_{32}^2)^{1/2}$ and $m_1 = (m_3^2 + \Delta m_{32}^2 - \Delta m_{21}^2)^{1/2}$.

Depending on the value of the lightest neutrino mass, m_{min} , the neutrino mass spectrum can be ($j = 1, 2, 3$):

- a) *Normal Hierarchical (NH)*: $m_1 \ll m_2 < m_3$, $m_2 \cong (\Delta m_{21}^2)^{1/2} \cong 8.68 \times 10^{-3}$ eV, $m_3 \cong (\Delta m_{31}^2)^{1/2} \cong 4.97 \times 10^{-2}$ eV or
- b) *Inverted Hierarchical (IH)*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m_{32}^2|^{1/2} \cong 4.97 \times 10^{-2}$ eV or
- c) *Quasi-Degenerate (QD)*: $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m_{31(32)}^2|$ and $m_0 \gtrsim 0.10$ eV.

If the mass of the lightest neutrino would turn out to be extremely small, say $m_{min} \lesssim 10^{-3}$, using the data on the neutrino oscillation parameters one finds that [7, 8] in the case of NH one has $|\langle m \rangle| \lesssim 0.005$ eV, while if the spectrum is IH, 0.01 eV $\lesssim |\langle m \rangle| \lesssim 0.05$ eV (see left panel in Fig. 1). A larger value of $|\langle m \rangle|$ up to approximately 0.5 eV is possible if the light neutrino mass spectrum is with partial hierarchy or is of quasi-degenerate type. In the latter case $|\langle m \rangle|$ can be close to the existing upper limits.

Isotope	$T_{1/2}^{0\nu}$ [10^{25} yr]	Experiment	$ \langle m \rangle $ [eV]
^{136}Xe	> 1.6	EXO-200	$0.16 - 0.30$
^{136}Xe	> 1.9	KamLAND-ZEN	$0.14 - 0.28$
^{76}Ge	> 2.1	GERDA	$0.23 - 0.54$
^{76}Ge	> 3.0	GERDA+IGEX+HdM	$0.19 - 0.45$

TABLE I. The experimental lower limits at 90% C.L. on the $(\beta\beta)_{0\nu}$ -decay half-lives of different isotopes and the corresponding lower and upper limits on $|\langle m \rangle|$ computed with NMEs calculated in the framework of different approaches (a summary table is given in [4]).

The most stringent upper limits on $|\langle m \rangle|$ were set by the IGEX (^{76}Ge), CUORICINO (^{130}Te), NEMO3 (^{100}Mo) and more recently by EXO-200, KamLAND-ZEN (^{136}Xe) and GERDA (^{76}Ge) experiments (see e.g. [9] for a summary). A lower limit on the half-life of ^{76}Ge , $T_{1/2}^{0\nu} > 1.9 \times 10^{25}$ yr (90% C.L.), was found in the Heidelberg-Moscow ^{76}Ge experiment (HdM) [10]. Further a positive $(\beta\beta)_{0\nu}$ -decay signal at $> 3\sigma$, corresponding to $T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|\langle m \rangle| = (0.1 - 0.9)$ eV, was claimed to have been observed in [11], and a later analysis reported evidence for $(\beta\beta)_{0\nu}$ -decay at 6σ corresponding to $|\langle m \rangle| = 0.32 \pm 0.03$ eV [12]. More recently, a large number of projects, or already running experiments, have aimed at a sensitivity of $|\langle m \rangle| \sim (0.01 - 0.05)$ eV, i.e., to probe the range of $|\langle m \rangle|$ corresponding to IH mass spectrum [9]: CUORE (^{130}Te), GERDA (^{76}Ge), SuperNEMO, EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), COBRA (^{116}Cd), XMASS (^{136}Xe), CANDLES (^{48}Ca), KamLAND-Zen (^{136}Xe), SNO+ (^{150}Nd), etc. Specifically among these, GERDA (Germanium Detector Array) at the Gran Sasso Laboratory (Italy) (^{76}Ge), EXO-200 (Enriched Xenon Observatory) in New Mexico (^{136}Xe) and KamLAND-Zen in Japan (^{136}Xe) are operational and they have been able to test the claim in [12]. In 2012 EXO-200 has obtained a lower limit on the half-life of ^{136}Xe [13],

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90\% C.L.}, \quad (4)$$

while later the experiment KamLAND-Zen reported the lower bound [14];

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yr at 90\% C.L.}. \quad (5)$$

In July 2013 also the GERDA collaboration reported the results from Phase I for $(\beta\beta)_{0\nu}$ decay of the isotope ^{76}Ge . No signal was observed and a lower limit has been set for $T_{1/2}^{0\nu}(^{76}\text{Ge})$ [15]:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90\% C.L.}, \quad (6)$$

which disprove, together with the other experimental results mentioned above, the claim in [12]. The GERDA collaboration reported also a combined limit using the bounds obtained by the HdM and IGEX experiments, that reads:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90\% C.L..} \quad (7)$$

In Table I we present the constraints on $|\langle m \rangle|$ obtained by the $(\beta\beta)_{0\nu}$ -decay experimental bounds mentioned above. In order to extract these constraints, we use as illustrative example, the sets of NMEs presented in Table 3 in [4] (see references therein for additional references on NMEs models) for the decays of the nuclides of interest, ^{76}Ge and ^{136}Xe .

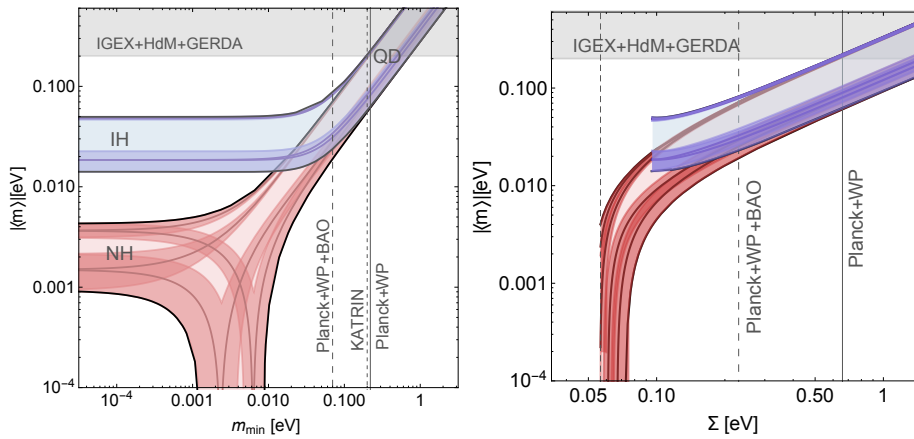


FIG. 1. Left Panel: The value of $|\langle m \rangle|$ as function of m_{\min} . The dotted line corresponds to the expected sensitivity of the future KATRIN β -decay experiment [16]. Right Panel: Values of $|\langle m \rangle|$ as function of the sum of the neutrino masses Σ . In both panels the light (strong) shaded regions indicate respectively the 3σ allowed CP-(non)conserving values of $|\langle m \rangle|$. In both plots the vertical solid and dashed lines are the constraints on Σ obtained by the Planck Collaboration [17]. The grey exclusion band is delimited by a value of $|\langle m \rangle| = 0.2$ eV, obtained using the 90 % C.L. limit on the half-life of ^{76}Ge reported in [15].

In Fig. 1 we show the predictions on $|\langle m \rangle|$ as function of m_{\min} (left panel) and as function of the sum of the neutrino masses Σ (right panel). We combined all the available data i.e we used the best fit values and the 3σ uncertainty of one of the most recent global fit analysis on the neutrino oscillation parameters [3], the results coming from $(\beta\beta)_{0\nu}$ -decays searches and the cosmological constraints from the Planck Collaboration [17]. The latter comes from the CMB data which yields an upper limit on the sum of the neutrino mass $\Sigma < 0.66 \text{ eV}$ at 95% C.L. (referred in Fig. 1 as “Planck + WP”), and the CMB data combined with the Barion Acoustic Oscillation data (BAO), for which $\Sigma < 0.23 \text{ eV}$ at 95% C.L. (referred as “Planck + WP + BAO”). In Fig. 1 we report also the expected sensitivity of the Karlsruhe TRitium Neutrino experiment (KATRIN) on the absolute scale of neutrino masses i.e. $m_{\bar{\nu}_e} \sim 0.2$ eV [1, 16], which is expected to start the data taking in late 2015.

From the left panel of Fig. 1, one can realize that, for $m_{\min} \lesssim 10^{-3}$ eV, in the case of NH spectrum, $\max|\langle m \rangle|$ is considerably smaller than $\min|\langle m \rangle|$ for the IH spectrum. This opens the possibility to obtain information about the neutrino mass pattern from a measurement of $|\langle m \rangle| \neq 0$ if $m_{\min} \lesssim 10^{-3}$ eV. More specifically, in the case $m_{\min} \lesssim 10^{-3}$ eV, a positive result in the future generation of $(\beta\beta)_{0\nu}$ -decay experiments with $|\langle m \rangle| \gtrsim 0.01$ eV, will imply that the NH spectrum is excluded (barring possible destructive interfering effects generated by new physics). If instead future searches will show that $|\langle m \rangle| \lesssim 0.01$ eV, both the IH and QD spectrum will be ruled out for massive Majorana neutrinos.

If in addition, it is established from oscillation experiments that $\Delta m_{31(32)}^2 < 0$, then one would deduce that either the neutrinos are Dirac fermions, or they are Majorana particles and there are additional contributions to $(\beta\beta)_{0\nu}$ -decay amplitude which interfere destructively [18].

Summarizing, the studies on $(\beta\beta)_{0\nu}$ -decay and a measurement of a nonzero value of $|\langle m \rangle| \geq$ (of a few 10^{-2} eV) could:

- establish the Majorana nature of massive neutrinos;
- give information on the type of neutrino mass spectrum. More specifically, a measured value of $|\langle m \rangle| \sim \text{few } 10^{-2} \text{ eV}$ can provide unique constraints on, or even can allow one to determine, the type of neutrino mass spectrum if neutrinos ν_i are Majorana particles [6–8, 19];
- provide also unique information on the absolute scale of neutrino masses or on the lightest neutrino mass (see e.g. [20]);
- with additional information from other sources (^3H β -decay experiments or cosmological and astrophysical data considerations) on the absolute neutrino mass scale, the $(\beta\beta)_{0\nu}$ -decay experiments can provide unique information on the Majorana CP-violation phases α_{31} and/or α_{21} [6, 21, 22].

If the $(\beta\beta)_{0\nu}$ -decay will be observed, the corresponding data will be used to constrain or even determining the possible mechanism(s) generating the decay. Indeed, the observation of the $(\beta\beta)_{0\nu}$ -decay would not guarantee that the dominant mechanism inducing the decay is the light Majorana neutrino exchange [23] (see also [24–27] for an example of how additional sterile neutrinos states affect the decay). The results of the $(\beta\beta)_{0\nu}$ -decay searches will play a very important role in testing and constraining i) theories of neutrino mass generation predicting massive Majorana neutrinos, and ii) the existence of new $|\Delta L| = 2$ couplings in the effective weak interaction Lagrangian, which could induce the decay. The existence of such couplings would have tremendous impact from the model-building point of view.

II. MULTIPLE MECHANISMS IN $(\beta\beta)_{0\nu}$ -DECAY

An observation of $(\beta\beta)_{0\nu}$ -decay would imply that the total lepton charge L is not conserved. This of course implies that the massive neutrinos get a Majorana mass [28, 29] and therefore are Majorana particles (see, e.g. [30]). However, the latter does not guarantee that the dominant mechanism inducing the $(\beta\beta)_{0\nu}$ -decay is the light Majorana neutrino exchange (that we will call the “standard” mechanism of the $(\beta\beta)_{0\nu}$ -decay) since the Majorana mass thus generated is exceedingly small. The $(\beta\beta)_{0\nu}$ -decay can well be due to the existence of interactions which do not conserve the total lepton charge L , specifically $\Delta L = \pm 2$. A number of such interactions have been proposed in the literature: heavy Majorana neutrinos coupled to the electron in the $V - A$ charged current weak interaction Lagrangian, supersymmetric (SUSY) theories with R -parity breaking terms which do not conserve the total lepton charge L , L -nonconserving couplings in the Left-Right symmetric theories, etc. At present we do not have evidence for the existence of $\Delta L \neq 0$ terms in the Lagrangian describing the particle interactions. Nevertheless, such terms can exist and they can be operative in the $(\beta\beta)_{0\nu}$ -decay. Moreover, it is impossible to exclude the hypothesis that, if observed, the $(\beta\beta)_{0\nu}$ -decay is triggered by more than one competing mechanisms.

The possibility of several different mechanisms contributing to the $(\beta\beta)_{0\nu}$ -decay amplitude was considered in [31] assuming that the corresponding $\Delta L = \pm 2$ couplings are CP conserving.

The analysis presented here is a natural continuation of the study performed in [31]. We consider the possibility of several different mechanisms contributing to the $(\beta\beta)_{0\nu}$ -decay amplitude in the general case of CP nonconservation: light Majorana neutrino exchange, heavy left-handed (LH) and heavy right-handed (RH) Majorana neutrino exchanges, lepton charge non-conserving couplings in SUSY theories with R -parity breaking. These different mechanisms can interfere only if the electron current

structure coincides and hence it can be factorized. If, on the contrary, these are not-interfering mechanisms, i.e., the electron currents have different chiralities, then the interference term is suppressed by a factor which depends on the considered nucleus. [32]. If the $(\beta\beta)_{0\nu}$ -decay is induced by, e.g., two “non-interfering” mechanisms (e.g. light Majorana neutrino and heavy RH Majorana neutrino exchanges), one can determine the absolute values of the two fundamental parameters, characterizing these mechanisms, from data on the half-lives of two nuclear isotopes. In the case when two “interfering” mechanisms are responsible for the $(\beta\beta)_{0\nu}$ -decay, the absolute values of the two relevant parameters and the interference term can be uniquely determined from data on the half-lives of three nuclei. We present in section IV illustrative examples of determination of the relevant fundamental parameters and of possible tests of the hypothesis that more than one mechanism is responsible for the $(\beta\beta)_{0\nu}$ -decay, using as input hypothetical half-lives of ^{76}Ge , ^{130}Te and ^{100}Mo and considering two “noninterfering” and two “interfering” mechanisms, namely, the light Majorana neutrino and the heavy RH Majorana neutrino exchanges, and the light Majorana neutrino and the dominant gluino exchanges. The effects of the uncertainties in the values of the nuclear matrix elements (NMEs) on the results of the indicated analyzes are also discussed and illustrated.

The method considered here can be generalized to the case of more than two $(\beta\beta)_{0\nu}$ -decay mechanisms. It has also the advantage that it allows to treat the cases of CP conserving and CP nonconserving couplings generating the $(\beta\beta)_{0\nu}$ -decay in a unique way. In section V we will investigate also the potential of combining data on one or more of the five nuclei ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe , for discriminating between different pairs of non-interfering or interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay.

III. POSSIBLE $\Delta L = 2$ COUPLINGS IN $(\beta\beta)_{0\nu}$ -DECAY

The $(\beta\beta)_{0\nu}$ -decay is allowed by a number of models, from the standard mechanism of light Majorana neutrino exchange to those such as Left-Right Symmetry [33, 34] or R-parity violating Supersymmetry (SUSY) [35]. These mechanisms might trigger $(\beta\beta)_{0\nu}$ -decay individually or together. The $(\beta\beta)_{0\nu}$ -decay half-life for a certain nucleus can therefore be written as function of some lepton number violating (LNV) parameters, each of them connected with a different mechanism i :

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}(E, Z) \left| \sum_i \eta_i^{LNV} \mathcal{M}_i^{0\nu} \right|^2 \quad (8)$$

where $G^{0\nu}(E, Z)$ and $\mathcal{M}_i^{0\nu}$ are, respectively, the phase-space factor (E_0 is the energy release) and the NMEs of the decay. The values of the phase space factor will be taken from [36]. Further, The NMEs depends on the mechanism generating the decay. Here, following the analysis in [36, 37], we will adopt the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA) [38, 39]. In this approach, the particle-particle strength parameter g_{pp} of the SRQRPA [40–42] is fixed by the recent data on the two-neutrino double beta decays of EXO-200. In the calculation of the $(\beta\beta)_{0\nu}$ -decay NMEs were considered the two-nucleon short-range correlations derived from same potential as residual interactions, namely from the Argonne or CD-Bonn potentials [43] and two values of the axial-vector constant are used, $g_A = 1.0, 1.25$. The numerical values used in the following analysis are taken from [36]. Let us make now some concrete examples on possible $(\beta\beta)_{0\nu}$ decay parameters. In the case of the light Majorana neutrino exchange mechanism of $(\beta\beta)_{0\nu}$ -decay, we can define a LNV parameter which is given by:

$$\eta_\nu = \frac{\langle m \rangle}{m_e} \quad (9)$$

where m_e is the electron mass. Next, we assume that the neutrino mass spectrum includes, in addition to the three light Majorana neutrinos, heavy Majorana states N_k with masses M_k much larger than the typical energy scale of the $(\beta\beta)_{0\nu}$ -decay, $M_k \gg 100$ MeV; we will consider the case of $M_k \gtrsim 10$ GeV. Such a possibility arises if the weak interaction Lagrangian includes right-handed (RH) sterile neutrino fields which couple to the LH flavour neutrino fields via the neutrino Yukawa coupling and possess a

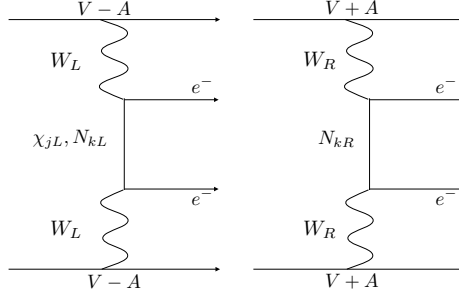


FIG. 2. Left Panel: Feynman diagrams for the $(\beta\beta)_{0\nu}$ -decay, generated by the light or heavy (LH) Majorana neutrino exchange (respectively χ_{jL} , and N_{kL}). Right Panel: the heavy (RH) Majorana neutrino exchange.

Majorana mass term. The heavy Majorana neutrinos N_k can mediate the $(\beta\beta)_{0\nu}$ -decay similar to the light Majorana neutrinos via the $V - A$ charged current weak interaction. The difference between the two mechanisms is that, unlike in the light Majorana neutrino exchange which leads to a long range inter-nucleon interactions, in the case of $M_k \gtrsim 10$ GeV of interest the momentum dependence of the heavy Majorana neutrino propagators can be neglected (i.e., the N_k propagators can be contracted to points) and, as a consequence, the corresponding effective nucleon transition operators are local. The LNV parameter in the case when the $(\beta\beta)_{0\nu}$ -decay is generated by the $(V - A)$ CC weak interaction due to the exchange of N_k can be written as:

$$\eta_L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}, \quad (10)$$

where m_p is the proton mass and U_{ek} is the element of the neutrino mixing matrix through which N_k couples to the electron in the weak charged lepton current. If the weak interaction Lagrangian contains also $(V + A)$ (i.e., right-handed (RH)) charged currents coupled to a RH charged weak boson W_R , as,

$$\mathcal{L}_{L+R} = \frac{g}{2\sqrt{2}} [(\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e)W_{\mu L}^- + (\bar{e}\gamma_\alpha(1 + \gamma_5)\nu_e)W_{\mu R}^-] \quad (11)$$

where $\nu_{eR} = \sum_k V_{ek} N_{kR}$, $C\bar{N}_k^T = \xi N_k$. Here V_{ek} are the elements of a mixing matrix by which N_k couple to the electron in the $(V + A)$ charged lepton current, M_L^W is the mass of the Standard Model charged weak boson, $M_L^W \cong 82$ GeV, and M_R^W is the mass of W_R . It follows from the existing data that [33, 34] $W_R \gtrsim 2.5$ TeV. For instance, in the $SU(2)_L \times SU(2)_R \times U(1)$ theories we can have also a contribution to the $(\beta\beta)_{0\nu}$ -decay amplitude generated by the exchange of virtual N_k coupled to the electron in the hypothetical $(V + A)$ CC part of the weak interaction Lagrangian. In this case the corresponding LNV parameter can be written as:

$$\eta_R = \left(\frac{M_L^W}{M_R^W} \right)^4 \sum_k^{heavy} V_{ek}^2 \frac{m_p}{M_k}. \quad (12)$$

If CP invariance does not hold, which we will assume to be the case in what follows, U_{ek} and V_{ek} will contain physical CP violating phases at least for some k and thus the parameters η_L and η_R will not be real. As can be shown, the NMEs corresponding to the two mechanisms of $(\beta\beta)_{0\nu}$ -decay with exchange of heavy Majorana neutrinos N_k , described in the present section, are the same and are given in [44]. We will denote them by $\mathcal{M}_N^{0\nu}$. Finally, it is important to note that the current factor in the $(\beta\beta)_{0\nu}$ -decay amplitude describing the two final state electrons, has different forms in the cases of $(\beta\beta)_{0\nu}$ -decay mediated by $(V - A)$ and $(V + A)$ CC weak interactions¹, namely, $\bar{e}(1 + \gamma_5)e^c \equiv 2\bar{e}_L(e^c)_R$

¹ The procedure is the same defined in the light neutrino exchange section. One has in this case $\bar{e}\gamma_\alpha P_R \gamma_\beta C\bar{e}^T A_{\alpha\beta} = \bar{e}P_L e^c A_{\alpha\beta}$

and $\bar{e}(1 - \gamma_5)e^c \equiv 2\bar{e}_R(e^c)_L$, respectively, where $e^c = C(\bar{e})^T$, C being the charge conjugation matrix. The difference in the chiral structure of the two currents leads to a relatively strong suppression of the interference between the terms in the $(\beta\beta)_{0\nu}$ -decay amplitude involving the two different electron current factors.

IV. UNCOVERING MULTIPLE CP NON-CONSERVING MECHANISMS

We are going to illustrate the possibility to get information about the different LNV parameters when two or more mechanisms are operative in $(\beta\beta)_{0\nu}$ -decay, analyzing the following two cases. First we consider two competitive “not-interfering” mechanisms of $(\beta\beta)_{0\nu}$ -decay: light left-handed Majorana neutrino exchange and heavy right-handed Majorana neutrino exchange. In this case the interference term arising in the $(\beta\beta)_{0\nu}$ -decay half-life from the product of the contributions due to the two mechanisms in the $(\beta\beta)_{0\nu}$ -decay amplitude, is strongly suppressed [32] as a consequence of the different chiral structure of the final state electron current in the two amplitudes. The latter leads to a different phase-space factor for the interference term, which is typically by a factor of 10 smaller than the standard one (corresponding to the contribution to the $(\beta\beta)_{0\nu}$ -decay half-life of each of the two mechanisms). More specifically, the suppression factors for ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te read, respectively [32]: 0.13; 0.08; 0.075 and 0.10. It is particularly small for ^{48}Ca : 0.04. In the analysis which follows we will neglect the contribution of the interference term in the $(\beta\beta)_{0\nu}$ -decay half-life. The effect of taking into account the interference term on the results thus obtained, as our numerical calculations have shown, does not exceed approximately 10%.

In the case of negligible interference term, the inverse value of the $(\beta\beta)_{0\nu}$ -decay half-life for a given isotope (A,Z) is given by:

$$\frac{1}{T_{1/2,i}^{0\nu} G_i^{0\nu}(E, Z)} \cong |\eta_\nu|^2 |\mathcal{M}_{i,\nu}^{0\nu}|^2 + |\eta_R|^2 |\mathcal{M}_{i,N}^{0\nu}|^2, \quad (13)$$

where the index i denotes the isotope. In the second illustrative case we consider $(\beta\beta)_{0\nu}$ -decay triggered by two active and “interfering” mechanisms: the light Majorana neutrino exchange and the LH Majorana heavy neutrino exchange. In this case, for a given nucleus, the inverse of the $(\beta\beta)_{0\nu}$ -decay half-life is given by:

$$\frac{1}{T_{1/2,i}^{0\nu} G_i^{0\nu}(E, Z)} = |\eta_\nu|^2 |\mathcal{M}_{i,\nu}^{0\nu}|^2 + |\eta_L|^2 |\mathcal{M}_{i,N}^{0\nu}|^2 + 2 \cos \alpha |\mathcal{M}_{i,N}^{0\nu}| |\mathcal{M}_{i,\nu}^{0\nu}| |\eta_\nu| |\eta_L|. \quad (14)$$

Here $|\eta_L|$ is the basic parameter of the LH heavy Neutrino exchange mechanism defined in eq. (10) and α is the relative phase of η_L and η_ν .

We will use in the following illustrative examples how one can extract information about $|\eta^{LNV}|$, using the lower limits on the $(\beta\beta)_{0\nu}$ -decay half-lives of ^{76}Ge , ^{82}Se and ^{100}Mo , and of ^{130}Te reported by the Heidelberg-Moscow [45], NEMO3 [46] and CUORICINO [47] experiments, respectively, as well as as well the ^{76}Ge half-life reported in [48] (see also [12]):

$$\begin{aligned} T_{1/2}^{0\nu}(^{76}\text{Ge}) &> 1.9 \times 10^{25} \text{ y} \text{ [45]} & T_{1/2}^{0\nu}(^{82}\text{Se}) &> 3.6 \times 10^{23} \text{ y} \text{ [46]}, \\ T_{1/2}^{0\nu}(^{100}\text{Mo}) &> 1.1 \times 10^{24} \text{ y} \text{ [46]}, & T_{1/2}^{0\nu}(^{130}\text{Te}) &> 3.0 \times 10^{24} \text{ y} \text{ [47]}. \\ T_{1/2}^{0\nu}(^{76}\text{Ge}) &= 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ y} \text{ [48]}. \end{aligned} \quad (15)$$

In the analysis which follows we will present numerical results first for $g_A = 1.25$ and using the NMEs calculated with the large size single particle basis (“large basis”) and the Charge Dependent Bonn (CD-Bonn) potential. Later results for $g_A = 1.0$, as well as for NMEs calculated with the Argonne potential, will also be reported.

A. Two “Non-Interfering” Mechanisms

In this case the solutions for the corresponding two LNV parameters $|\eta_A|^2$ and $|\eta_B|^2$ obtained from data on the $(\beta\beta)_{0\nu}$ -decay half-lives of the two isotopes (A_i, Z_i) and (A_j, Z_j) , are given by:

$$|\eta_A|^2 = \frac{|\mathcal{M}_{j,B}^{0\nu}|^2/T_i G_i - |\mathcal{M}_{i,B}^{0\nu}|^2/T_j G_j}{|\mathcal{M}_{i,A}^{0\nu}|^2|\mathcal{M}_{j,B}^{0\nu}|^2 - |\mathcal{M}_{i,B}^{0\nu}|^2|\mathcal{M}_{j,A}^{0\nu}|^2}, \quad |\eta_B|^2 = \frac{|\mathcal{M}_{i,A}^{0\nu}|^2/T_j G_j - |\mathcal{M}_{j,A}^{0\nu}|^2/T_i G_i}{|\mathcal{M}_{i,A}^{0\nu}|^2|\mathcal{M}_{j,B}^{0\nu}|^2 - |\mathcal{M}_{i,B}^{0\nu}|^2|\mathcal{M}_{j,A}^{0\nu}|^2}. \quad (16)$$

It follows from eq. (16) that if one of the two half-lives, say T_i , is fixed, the positivity conditions $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$ can be satisfied only if T_j lies in a specific “positivity interval”. Choosing for convenience always $A_j < A_i$ we get for the positivity interval:

$$\frac{G_i}{G_j} \frac{|\mathcal{M}_{i,B}^{0\nu}|^2}{|\mathcal{M}_{j,B}^{0\nu}|^2} T_i^{exp} \leq \frac{G_i}{G_j} \frac{|\mathcal{M}_{i,B}^{0\nu}|^2}{|\mathcal{M}_{j,B}^{0\nu}|^2} T_i \leq T_j \leq \frac{G_i}{G_j} \frac{|\mathcal{M}_{i,A}^{0\nu}|^2}{|\mathcal{M}_{j,A}^{0\nu}|^2} T_i, \quad (17)$$

where we have used $|\mathcal{M}_{i,A}^{0\nu}|^2/|\mathcal{M}_{j,A}^{0\nu}|^2 > |\mathcal{M}_{i,B}^{0\nu}|^2/|\mathcal{M}_{j,B}^{0\nu}|^2$ and the first inequality in eq. (17) has been obtained considering the lower bound of interest for the isotope (A_i, Z_i) , i.e., if $T_i \geq T_i^{exp}$. If only one of the mechanism is present, e.g. the light neutrino exchange mechanism, and the NME are correctly calculated, the $(\beta\beta)_{0\nu}$ -decay effective Majorana mass (and $|\eta_\nu|^2$) extracted from all three (or any number of) $(\beta\beta)_{0\nu}$ -decay isotopes must be the same (see, e.g., [20, 49]). Similarly, if the heavy RH Majorana neutrino exchange gives the dominant contribution, the extracted value $|\eta_R|^2$ must be the same for all three (or more) $(\beta\beta)_{0\nu}$ -decay nuclei. Assuming $\eta_A \equiv \eta_\nu$ and $\eta_B \equiv \eta_R$ and using the values of the phase-space factors and the two relevant NME for “CD-Bonn, large, $g_A = 1.25(1.0)$ ”, we get for example:

$$0.15 \leq \frac{T(^{100}Mo)}{T(^{76}Ge)} \leq 0.18(0.17), 0.17 \leq \frac{T(^{130}Te)}{T(^{76}Ge)} \leq 0.22(0.23), 1.14(1.16) \leq \frac{T(^{130}Te)}{T(^{100}Mo)} \leq 1.24(1.30). \quad (18)$$

It is quite remarkable that the physical solutions are possible only if the ratio of the half-lives of all the pairs of the three isotopes considered take values in very narrow intervals. This result is a consequence of the values of the phase space factors and of the NME for the two mechanisms considered. In the case of the Argonne potential, “large basis” and $g_A = 1.25$ (1.0) we get very similar results:

$$0.15 \leq \frac{T(^{100}Mo)}{T(^{76}Ge)} \leq 0.18, 0.18 \leq \frac{T(^{130}Te)}{T(^{76}Ge)} \leq 0.24 (0.25), 1.22 \leq \frac{T(^{130}Te)}{T(^{100}Mo)} \leq 1.36 (1.42). \quad (19)$$

If it is experimentally established that any of the three ratios of half-lives considered lies outside the interval of physical solutions of $|\eta_\nu|^2$ and $|\eta_R|^2$, obtained taking into account all relevant uncertainties, one would be led to conclude that the $(\beta\beta)_{0\nu}$ -decay is not generated by the two mechanisms under discussion. In order to show that the constraints given above are indeed satisfied, the relevant ratios of $(\beta\beta)_{0\nu}$ -decay half-lives should be known with a remarkably small uncertainty (not exceeding approximately 5% of the central values of the intervals).

Obviously, given the half-life of one isotope, constraints similar to those described above can be derived on the half-life of any other isotope beyond those considered by us. Similar constraints can be obtained in all cases of two “non-interfering” mechanisms generating the $(\beta\beta)_{0\nu}$ -decay. The predicted intervals of half-lives of the various isotopes will differ, in general, for the different pairs of “non-interfering” mechanisms. However, as it is shown [37], these differences in the cases of the $(\beta\beta)_{0\nu}$ -decay triggered by the exchange of heavy Majorana neutrinos coupled to (V+A) currents and i) the gluino exchange mechanism, or ii) the squark-neutrino exchange mechanism, are extremely small. One of the consequences of this feature of the different pairs of “non-interfering” mechanisms considered by us is that if it will be possible to rule out one of them as the cause of $(\beta\beta)_{0\nu}$ -decay, most likely one will be able to rule out all three of them. The set of constraints under discussion will not be valid, in general, if the $(\beta\beta)_{0\nu}$ -decay is triggered by two “interfering” mechanisms with a

TABLE II. The predictions for the half-life of a third nucleus (A_3, Z_3) , using as input in the equations for $|\eta_\nu|^2$ and $|\eta_R|^2$, eq. (16), the half-lives of two other nuclei (A_1, Z_1) and (A_2, Z_2) . The three nuclei used are ^{76}Ge , ^{100}Mo and ^{130}Te . The results shown are obtained for a fixed value of the half-life of (A_1, Z_1) and assuming the half-life of (A_2, Z_2) to lie in a certain specific interval. The physical solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ and then used to derive predictions for the half-life of the third nucleus (A_3, Z_3) . The latter are compared with the lower limits given in eq. (15). The results quoted are obtained for NMEs given in the columns “CD-Bonn, large, $g_A = 1.25$ ” in [36]. One star beside the isotope pair whose half-lives are used as input for the system of equations (16), indicates predicted ranges of half-lives of the nucleus (A_3, Z_3) that are not compatible with the lower bounds given in (15).

Pair	$T_{1/2}^{0\nu}(A_1, Z_1)$ [yr]	$T_{1/2}^{0\nu}(A_2, Z_2)$ [yr]	Prediction on $[A_3, Z_3]$ [yr]
$^{76}\text{Ge} - ^{100}\text{Mo}$	$T(\text{Ge}) = 2.23 \cdot 10^{25}$	$3.23 \cdot 10^{24} \leq T(\text{Mo}) \leq 3.97 \cdot 10^{24}$	$3.68 \cdot 10^{24} \leq T(\text{Te}) \leq 4.93 \cdot 10^{24}$
$^{76}\text{Ge} - ^{130}\text{Te}$	$T(\text{Ge}) = 2.23 \cdot 10^{25}$	$3.68 \cdot 10^{24} \leq T(\text{Te}) \leq 4.93 \cdot 10^{24}$	$3.23 \cdot 10^{24} \leq T(\text{Mo}) \leq 3.97 \cdot 10^{24}$
$^{100}\text{Mo} - ^{130}\text{Te} \star$	$T(\text{Mo}) = 5.8 \cdot 10^{23}$	$6.61 \cdot 10^{23} \leq T(\text{Te}) \leq 7.20 \cdot 10^{23}$	$3.26 \cdot 10^{24} \leq T(\text{Ge}) \leq 4.00 \cdot 10^{24}$
$^{76}\text{Ge} - ^{100}\text{Mo}$	$T(\text{Ge}) = 10^{26}$	$1.45 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.78 \cdot 10^{25}$	$1.65 \cdot 10^{25} \leq T(\text{Te}) \leq 2.21 \cdot 10^{25}$
$^{76}\text{Ge} - ^{130}\text{Te}$	$T(\text{Ge}) = 10^{26}$	$1.65 \cdot 10^{25} \leq T(\text{Te}) \leq 2.21 \cdot 10^{25}$	$1.45 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.78 \cdot 10^{25}$
$^{100}\text{Mo} - ^{130}\text{Te}$	$T(\text{Mo}) = 5.8 \cdot 10^{24}$	$6.61 \cdot 10^{24} \leq T(\text{Te}) \leq 7.20 \cdot 10^{24}$	$3.26 \cdot 10^{25} \leq T(\text{Ge}) \leq 4.00 \cdot 10^{25}$
$^{100}\text{Mo} - ^{130}\text{Te} \star$	$T(\text{Te}) = 3 \cdot 10^{24}$	$2.42 \cdot 10^{24} \leq T(\text{Mo}) \leq 2.63 \cdot 10^{24}$	$1.36 \cdot 10^{25} \leq T(\text{Ge}) \leq 1.82 \cdot 10^{25}$
$^{100}\text{Mo} - ^{130}\text{Te}$	$T(\text{Te}) = 1.65 \cdot 10^{25}$	$1.33 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.45 \cdot 10^{25}$	$7.47 \cdot 10^{25} \leq T(\text{Ge}) \leq 1.00 \cdot 10^{26}$
$^{100}\text{Mo} - ^{130}\text{Te}$	$T(\text{Te}) = 3 \cdot 10^{25}$	$2.42 \cdot 10^{25} \leq T(\text{Mo}) \leq 2.63 \cdot 10^{25}$	$1.36 \cdot 10^{26} \leq T(\text{Ge}) \leq 1.82 \cdot 10^{26}$

non-negligible interference term, or by more than two mechanisms with significant contributions to the $(\beta\beta)_{0\nu}$ -decay rates of the different nuclei.

We analyze next the possible solutions for different combinations of the half-lives of the following isotopes: ^{76}Ge , ^{100}Mo and ^{130}Te . Assuming the half-lives of two isotopes to be known and using the physical solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ obtained using these half-lives, one can obtain a prediction for the half-life of the third isotope. The predicted half-life should satisfy the existing lower limits on it. In the calculations the results of which are reported here, we fixed the half-life of one of the two isotopes and assumed the second half-life lies in an interval compatible with the existing constraints. We used the value of $T_{1/2}^{0\nu}(^{76}\text{Ge})$ and values of $T_{1/2}^{0\nu}(^{100}\text{Mo})$ and $T_{1/2}^{0\nu}(^{130}\text{Te})$ from the intervals given in (15). The system of two equations is solved and the values of $|\eta_\nu|^2 > 0$ and $|\eta_R|^2 > 0$ thus obtained were used to obtain predictions for the half-life of the third isotope. The results for NMEs corresponding to the case “CD-Bonn, large, $g_A = 1.25$ ” are given in Table II. We note that the experimental lower bounds quoted in eq. (15) have to be taken into account since they can further constrain the range of allowed values of $|\eta_\nu|^2$ and $|\eta_R|^2$. Indeed, an inspection of the values in Table II shows that not all the ranges predicted for the third half-life using the solutions obtained for $|\eta_R|^2$ and $|\eta_\nu|^2$ are compatible with the lower bounds on the half-life of the considered nuclear isotopes, given in (15). In this case, some or all “solution” values of $|\eta_R|^2$ and/or $|\eta_\nu|^2$ are ruled out. In Table II these cases are marked by a star.

The results reported in Table II are stable with respect to variations of the NMEs. If we use the NMEs corresponding to the case “CD-Bonn, large, $g_A = 1.0$ ”, the limits of the intervals quoted in Table II change by $\pm 5\%$. If instead we use the NMEs corresponding to the Argonne potential, “large basis” and $g_A = 1.25$ ($g_A = 1.0$), the indicated limits change by $\pm 10\%$ ($\pm 14\%$).

V. LARGELY DIFFERENT NMES AND $(\beta\beta)_{0\nu}$ -DECAY

The observation of $(\beta\beta)_{0\nu}$ -decay of several different isotopes is crucial for obtaining information about the mechanism or mechanisms that induce the decay. In this section we investigate the possibility to discriminate between different pairs of CP non-conserving mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay by using data on $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different NMEs. In addition to the nuclei ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te we will employ also the isotope ^{136}Xe . Four sets of nuclear matrix elements (NMEs) of the decays of these five nuclei, derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA), will be employed in our analysis. The analysis we are

going to present is based on the fact that for each of the five single mechanisms discussed in [36], the NMEs for ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te differ relatively little —being the relative difference between the NMEs of any two nuclei not exceeding 10%. The NMEs for ^{136}Xe instead differ significantly from those of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , being by a factor $\sim (1.3 - 2.5)$ smaller. This allows, in principle, to draw conclusions about the pair of non-interfering (interfering) mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and of at least one of the other used isotopes. We will employ the lower bound obtained by the EXO collaboration on the $(\beta\beta)_{0\nu}$ -decay half-life of ^{136}Xe [13]:

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{y} \quad (90 \% \text{ CL}). \quad (20)$$

Suppose now we analyse the case of two non-interfering mechanisms i.e. that $T_i \equiv T_{1/2}^{0\nu}(^{136}\text{Xe})$, $T_j \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$ and that the $(\beta\beta)_{0\nu}$ -decay is due by the standard light neutrino exchange and the heavy RH Majorana neutrino exchange. In this case the positivity conditions for $|\eta_\nu|^2$ and $|\eta_R|^2$ imply for the Argonne [and CD-Bonn] NMEs corresponding to $g_A = 1.25$ (1.0):

$$1.90 (1.85) [1.30 (1.16)] \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.70 (2.64), [2.47 (2.30)] ; \quad (21)$$

Using the EXO result, eq. (20), and the Argonne NMEs we get the lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 3.03 (2.95) \times 10^{25} \text{y}. \quad (22)$$

This lower bound is significantly bigger than the experimental lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$ quoted in eq. (15). If we use instead the CD-Bonn NMEs, the limit we obtain is close to the experimental lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 2.08 (1.85) \times 10^{25} \text{y}. \quad (23)$$

For illustrative purposes we show in Fig. 3 the solutions of equation (16) for $|\eta_\nu|^2$ and $|\eta_R|^2$ derived by fixing $T_{1/2}^{0\nu}(^{76}\text{Ge})$ to the best fit value claimed in [48], $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ (see eq. (15)). As Fig. 3 shows, the positive (physical) solutions obtained using the Argonne NMEs are incompatible with the EXO result, eq. (20), and under the assumptions made and according to our oversimplified analysis, are ruled out. At the same time, the physical solutions obtained using the CD-Bonn NMEs are compatible with the EXO limit for values of $|\eta_\nu|^2$ and $|\eta_R|^2$ lying in a relatively narrow interval.

As we noticed in [37], if the experimentally determined interval of allowed values of the ratio T_j/T_i of the half-lives of the two isotopes considered, including all relevant uncertainties, lies outside the range of positive solutions for $|\eta_A|^2$ and $|\eta_B|^2$, one would be led to conclude that the $(\beta\beta)_{0\nu}$ -decay is not generated by the two mechanisms under discussion.

VI. DISCRIMINATING BETWEEN DIFFERENT PAIRS OF NON-INTERFERING MECHANISMS

The first thing to notice is that for each of the four different mechanisms of $(\beta\beta)_{0\nu}$ -decay considered, the relative difference between NMEs of the decays of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te does not exceed approximately 10%: $(\mathcal{M}_{j,X}^{0\nu} - \mathcal{M}_{i,X}^{0\nu}) / (0.5(\mathcal{M}_{j,X}^{0\nu} + \mathcal{M}_{i,X}^{0\nu})) \lesssim 0.1$, where $i \neq j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$, and X denotes any one of the four mechanisms discussed. As was shown in the previous section this leads to degeneracies between the positivity intervals of values of the ratio of the half-lives of any two given of the indicated four isotopes, corresponding to the different pairs of mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay. The degeneracies in question make it practically impossible to distinguish between the different pairs of $(\beta\beta)_{0\nu}$ -decay mechanisms, considered in the previous section and in the present section, using data on the half-lives of two or more of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . At

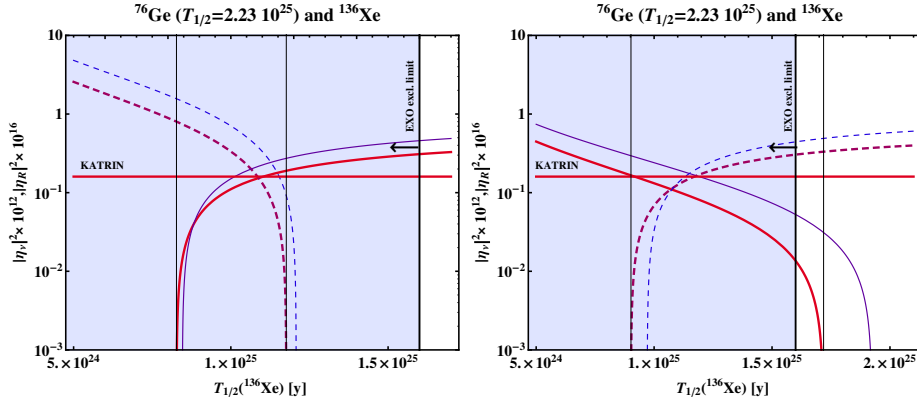


FIG. 3. The values of $|\eta_\nu|^2$ (solid lines) and $|\eta_R|^2$ (dashed lines) obtained for $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y [48] as a function of $T_{1/2}^{0\nu}(^{136}\text{Xe})$, using the Argonne (left panel) and CD-Bonn (right panel) NMEs corresponding to $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines). The region of physical (positive) solutions for $g_A = 1.25$ are delimited by the two vertical lines. The solid horizontal line corresponds to the prospective upper limit from the KATRIN experiment [16], while the thick solid vertical line indicates the EXO lower bound [13]. The gray areas correspond to excluded values of $|\eta_\nu|^2$ and $|\eta_R|^2$.

the same time, it is possible, in principle, to exclude them all using data on the half-lives of at least two of the indicated four nuclei. In contrast, the NMEs for the $(\beta\beta)_{0\nu}$ -decay of ^{136}Xe , corresponding to each of the four different mechanisms we are considering are by a factor of $\sim (1.3 - 2.5)$ smaller than the $(\beta\beta)_{0\nu}$ -decay NMEs of the other four isotopes listed above: $(\mathcal{M}_{j,X}^{0\nu} - \mathcal{M}_{i,X}^{0\nu})/\mathcal{M}_{i,X}^{0\nu} \cong (0.3 - 1.5)$, where $i = ^{136}\text{Xe}$ and $j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$ (see Figs. 4). As a consequence, using data on the half-life of ^{136}Xe as input in determining the positivity interval of values of the half-life of any second isotope lifts to a certain degree the degeneracy of the positivity intervals corresponding to different pairs of non-interfering mechanisms. This allows, in principle, to draw conclusions about the pair of mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and a second isotope which can be, e.g., any of the four considered above, ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te .

Therefore, we analyze next the possibility to discriminate between two pairs of non-interfering mechanisms triggering the $(\beta\beta)_{0\nu}$ -decay when the pairs share one mechanism. Given three different non-interfering mechanisms A , B and C , we can test the hypothesis of the $(\beta\beta)_{0\nu}$ -decay induced by the pairs i) $A + B$ or ii) $C + B$, using the half-lives of the same two isotopes. As a consequence of the fact that B is common to both pairs of mechanisms, the numerators of the expressions for $|\eta_A|^2$ and $|\eta_C|^2$, as it follows from eq. (16), coincide. Correspondingly, using the half-lives of the same two isotopes would allow us to distinguish, in principle, between the cases i) and ii) if the denominators in the expressions for the solutions for $|\eta_A|^2$ and $|\eta_C|^2$ have opposite signs. Indeed, in this case the physical solutions for $|\eta_A|^2$ in the case i) and $|\eta_C|^2$ in the case ii) will lie either in the positivity intervals, see e.g. eq. (17). Thus, the positivity solution intervals for $|\eta_A|^2$ and $|\eta_C|^2$ would not overlap, except for the point corresponding to a value of the second isotope half-life where $\eta_A = \eta_C = 0$. This would allow, in principle, to discriminate between the two considered pairs of mechanisms.

It follows from the preceding discussion that in order to be possible to discriminate between the pairs $A + B$ and $C + B$ of non-interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay, the following condition has to be fulfilled:

$$\frac{\text{Det} \begin{pmatrix} |\mathcal{M}_{i,A}^{0\nu}|^2 & |\mathcal{M}_{i,B}^{0\nu}|^2 \\ |\mathcal{M}_{j,A}^{0\nu}|^2 & |\mathcal{M}_{j,B}^{0\nu}|^2 \end{pmatrix}}{\text{Det} \begin{pmatrix} |\mathcal{M}_{i,C}^{0\nu}|^2 & |\mathcal{M}_{i,B}^{0\nu}|^2 \\ |\mathcal{M}_{j,C}^{0\nu}|^2 & |\mathcal{M}_{j,B}^{0\nu}|^2 \end{pmatrix}} = \frac{|\mathcal{M}_{i,A}^{0\nu}|^2 |\mathcal{M}_{j,B}^{0\nu}|^2 - |\mathcal{M}_{i,B}^{0\nu}|^2 |\mathcal{M}_{j,A}^{0\nu}|^2}{|\mathcal{M}_{i,C}^{0\nu}|^2 |\mathcal{M}_{j,B}^{0\nu}|^2 - |\mathcal{M}_{i,B}^{0\nu}|^2 |\mathcal{M}_{j,C}^{0\nu}|^2} < 0. \quad (24)$$

This condition is satisfied if one of the following two sets of inequalities holds:

$$I) \frac{\mathcal{M}_{j,C}^{0\nu} - \mathcal{M}_{i,C}^{0\nu}}{\mathcal{M}_{i,C}^{0\nu}} < \frac{\mathcal{M}_{j,B}^{0\nu} - \mathcal{M}_{i,B}^{0\nu}}{\mathcal{M}_{i,B}^{0\nu}} < \frac{\mathcal{M}_{j,A}^{0\nu} - \mathcal{M}_{i,A}^{0\nu}}{\mathcal{M}_{i,A}^{0\nu}}, \quad (25)$$

$$II) \frac{\mathcal{M}_{j,A}^{0\nu} - \mathcal{M}_{i,A}^{0\nu}}{\mathcal{M}_{i,A}^{0\nu}} < \frac{\mathcal{M}_{j,B}^{0\nu} - \mathcal{M}_{i,B}^{0\nu}}{\mathcal{M}_{i,B}^{0\nu}} < \frac{\mathcal{M}_{j,C}^{0\nu} - \mathcal{M}_{i,C}^{0\nu}}{\mathcal{M}_{i,C}^{0\nu}}. \quad (26)$$

One example of a possible application of the preceding results is provided by the mechanisms of light Majorana neutrino exchange (A), RH heavy Majorana neutrino exchange (B) and gluino exchange (C) and the Argonne NMEs. We are interested in studying cases involving ^{136}Xe since, as it was already discussed earlier, the NMEs of ^{136}Xe differ significantly from those of the lighter isotopes such as ^{76}Ge . Indeed, as can be shown, it is possible, in principle, to discriminate between the two pairs $A + B$ and $B + C$ of the three mechanisms indicated above if we combine data on the half-life of ^{136}Xe with those on the half-life of one of the four isotopes ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , and use the Argonne NMEs in the analysis. In this case the inequalities (25) are realized, as can be seen in the left panel of Fig. 4, where we plot the relative differences $(\mathcal{M}_j^{0\nu} - \mathcal{M}_i^{0\nu})/\mathcal{M}_i^{0\nu}$ for the Argonne NMEs where the indices i and j refer respectively to ^{136}Xe and to one of the four isotopes ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . In the case of the CD-Bonn NMEs (right panel of Fig. 4), the inequalities (25) or (26) do not hold for the pairs of mechanisms considered. The inequalities given in eq. (25) hold, as it follows from right panel of Fig. 4, if, e.g., the mechanisms A, B and C are respectively the heavy RH Majorana neutrino exchange, the light Majorana neutrino exchange and the gluino exchange (for a definition of the gluino exchange mechanism see [36, 37], and references therein). In the following we will denote the latter with $\eta_{\lambda'}$.

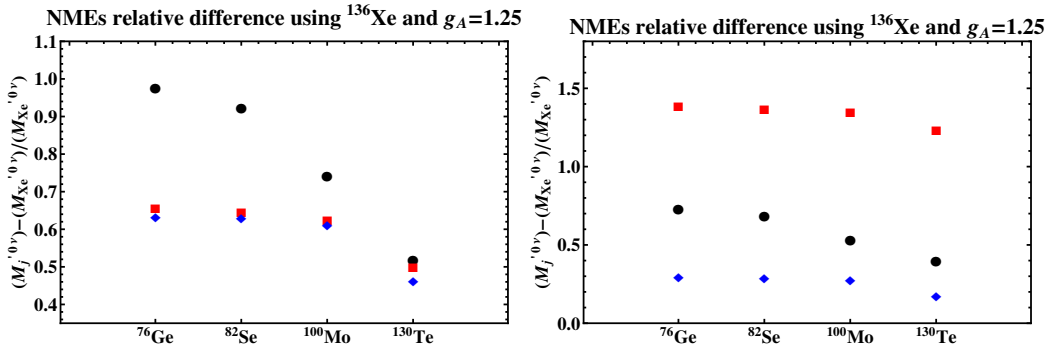


FIG. 4. The relative differences between the Argonne and Cd-Bonn NMEs $(\mathcal{M}_j^{0\nu} - \mathcal{M}_i^{0\nu})/\mathcal{M}_i^{0\nu}$, where $i = ^{136}\text{Xe}$ and $j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$, for $g_A = 1.25$ and for three different non-interfering mechanisms: light Majorana neutrino exchange (circles), RH heavy Majorana neutrino exchange (squares) and gluino exchange (diamonds). See text for details.

The preceding considerations are illustrated graphically in Figs. 5 and 6. In Fig. 5 we use $T_i \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$ and $T_j \equiv T_{1/2}^{0\nu}(^{136}\text{Xe})$ and the Argonne (left panel) and CD-Bonn (right panel) NMEs for the decays of ^{76}Ge and ^{136}Xe to show the possibility of discriminating between the two pairs of non-interfering mechanisms considered earlier: i) light Majorana neutrino exchange and heavy RH Majorana neutrino exchange (RHN) and ii) heavy RH Majorana neutrino exchange and gluino exchange. The ^{76}Ge half-life is set to $T_i = 5 \times 10^{25}$ y, while that of ^{136}Xe , T_j , is allowed to vary in a certain interval. The solutions for the three LNV parameters corresponding to the three mechanisms considered, $|\eta_\nu|^2$, $|\eta_R|^2$ and $|\eta_{\lambda'}|^2$, obtained for the chosen value of T_i and interval of values of T_j , are shown as functions of T_j . As is clearly seen in the left panel of Fig. 5, if $|\eta_\nu|^2$, $|\eta_R|^2$ and $|\eta_{\lambda'}|^2$ are obtained using the Argonne NMEs, the intervals of values of T_j for which one obtains the physical positive solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$, do not overlap. This makes it possible, in principle, to determine which of the two pairs of mechanisms considered (if any) is inducing the $(\beta\beta)_{0\nu}$ -decay. The same result does not hold if one uses the CD-Bonn NMEs in the analysis, as is illustrated in the right panel

of Fig. 5. In this case none of the inequalities (25) and (26) is fulfilled, the intervals of values of T_j for which one obtains physical solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$ overlap and the discrimination between the two pairs of mechanisms is problematic.

We show in Fig. 6 that the features of the solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$ we have discussed above, which are related to the values of the relevant NMEs, do not change if one uses in the analysis the half-lives and NMEs of ^{136}Xe and of another lighter isotope instead of ^{76}Ge , namely, of ^{100}Mo .

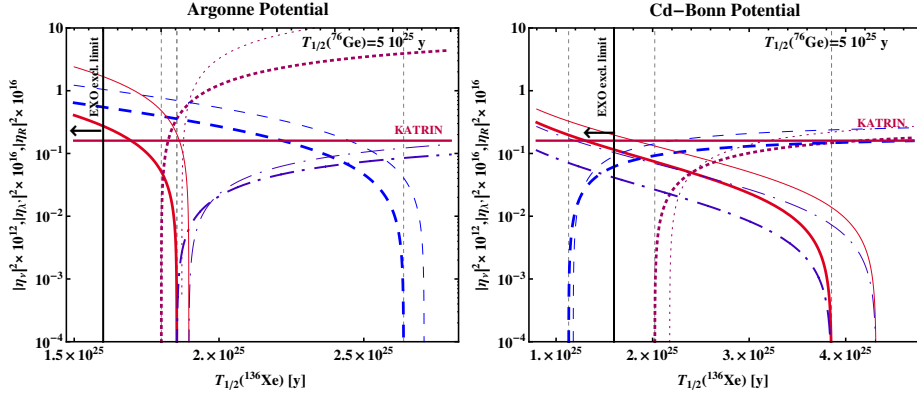


FIG. 5. Solutions for the LNV parameters corresponding to two pairs of non-interfering mechanisms: i) $|\eta_\nu|^2$ and $|\eta_R|^2$ (dot-dashed and dashed lines) and ii) $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ (solid and dotted lines). The solutions are obtained by fixing $T_i = T_{1/2}^{0\nu}(^{76}\text{Ge}) = 5 \times 10^{25}$ y and letting free $T_j = T_{1/2}^{0\nu}(^{136}\text{Xe})$ and using the sets of Argonne (left panel) and CD-Bonn (right panel) NMEs calculated for $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines). The range of positive solutions in the case of Argonne NMEs and $g_A = 1.25$ is delimited by the two vertical dashed lines.

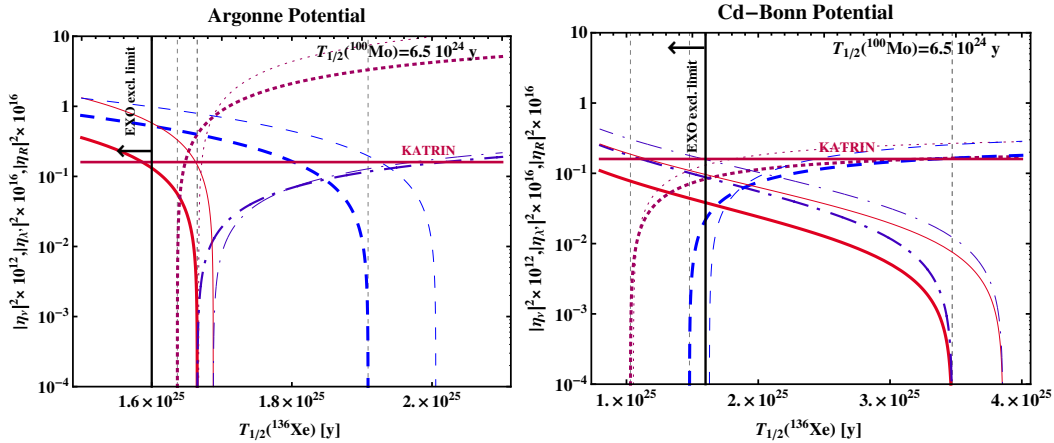


FIG. 6. Solutions for the LNV parameters of two pairs of non-interfering $(\beta\beta)_{0\nu}$ -decay mechanisms i) $|\eta_\nu|^2$ and $|\eta_R|^2$ (dot-dashed and dashed lines) and ii) $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ (solid and dotted lines) obtained by fixing $T_i = T_{1/2}^{0\nu}(^{100}\text{Mo}) = 6.5 \times 10^{24}$ yr and letting free $T_j = T_{1/2}^{0\nu}(^{136}\text{Xe})$.

VII. TWO INTERFERING MECHANISMS

We analyze in the present Section the possibility of $(\beta\beta)_{0\nu}$ -decay induced by two interfering CP-non-conserving mechanisms. As we have seen in the previous section this case is characterized by three parameters: the absolute values and the relative phase of the two LNV parameters associated

with the two mechanisms. They can be determined, in principle, from data on the half-lives of three isotopes, T_i , $i = 1, 2, 3$. Given $T_{1,2,3}$ and denoting by A and B the two mechanisms, one can set a system of three linear equations in three unknowns, the solution of which reads:

$$|\eta_A|^2 = \frac{D_i}{D}, \quad |\eta_B|^2 = \frac{D_j}{D}, \quad z \equiv 2 \cos \alpha |\eta_A| |\eta_B| = \frac{D_k}{D}, \quad (27)$$

where D , D_i , D_j and D_k are the following determinants:

$$D = \begin{vmatrix} (\mathcal{M}_{i,A}^{0\nu})^2 & (\mathcal{M}_{i,B}^{0\nu})^2 & \mathcal{M}_{i,B}^{0\nu} \mathcal{M}_{i,A}^{0\nu} \\ (\mathcal{M}_{j,A}^{0\nu})^2 & (\mathcal{M}_{j,B}^{0\nu})^2 & \mathcal{M}_{j,B}^{0\nu} \mathcal{M}_{j,A}^{0\nu} \\ (\mathcal{M}_{k,A}^{0\nu})^2 & (\mathcal{M}_{k,B}^{0\nu})^2 & \mathcal{M}_{k,B}^{0\nu} \mathcal{M}_{k,A}^{0\nu} \end{vmatrix}, \quad D_i = \begin{vmatrix} i/T_i G_i & (\mathcal{M}_{i,B}^{0\nu})^2 & \mathcal{M}_{i,B}^{0\nu} \mathcal{M}_{i,A}^{0\nu} \\ i/T_j G_j & (\mathcal{M}_{j,B}^{0\nu})^2 & \mathcal{M}_{j,B}^{0\nu} \mathcal{M}_{j,A}^{0\nu} \\ i/T_k G_k & (\mathcal{M}_{k,B}^{0\nu})^2 & \mathcal{M}_{k,B}^{0\nu} \mathcal{M}_{k,A}^{0\nu} \end{vmatrix}, \quad (28)$$

$$D_j = \begin{vmatrix} (\mathcal{M}_{i,A}^{0\nu})^2 & i/T_i G_i & \mathcal{M}_{i,B}^{0\nu} \mathcal{M}_{i,A}^{0\nu} \\ (\mathcal{M}_{j,A}^{0\nu})^2 & i/T_j G_j & \mathcal{M}_{j,B}^{0\nu} \mathcal{M}_{j,A}^{0\nu} \\ (\mathcal{M}_{k,A}^{0\nu})^2 & i/T_k G_k & \mathcal{M}_{k,B}^{0\nu} \mathcal{M}_{k,A}^{0\nu} \end{vmatrix}, \quad D_k = \begin{vmatrix} (\mathcal{M}_{i,A}^{0\nu})^2 & (\mathcal{M}_{i,B}^{0\nu})^2 & i/T_i G_i \\ (\mathcal{M}_{j,A}^{0\nu})^2 & (\mathcal{M}_{j,B}^{0\nu})^2 & i/T_j G_j \\ (\mathcal{M}_{k,A}^{0\nu})^2 & (\mathcal{M}_{k,B}^{0\nu})^2 & i/T_k G_k \end{vmatrix}. \quad (29)$$

As in the case of two non-interfering mechanisms, the LNV parameters must be non-negative $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$, and in addition the interference term must satisfy the following condition:

$$-2|\eta_A| |\eta_B| \leq 2 \cos \alpha |\eta_A| |\eta_B| \leq 2|\eta_A| |\eta_B|. \quad (30)$$

These conditions will be called from here on “positivity conditions”.

Using the positivity conditions it is possible to determine the interval of positive solutions for one of the three half-life, e.g., T_k , if the values of the other two half-lives in the equations have been measured and are known. The condition on the interference term in equation (17) can considerably reduce the interval of values of T_k where $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$. In Table III we give examples of the constraints on T_k following from the positivity conditions for three different pairs of interfering mechanisms: light Majorana neutrino and supersymmetric gluino exchange; light Majorana neutrino exchange and heavy LH Majorana neutrino exchange; gluino exchange and heavy LH Majorana neutrino exchange. It follows from the results shown in Table III, in particular, that when $T_{1/2}^{0\nu}(^{76}\text{Ge})$ is set to $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}; 10^{26}$ y, but $T_{1/2}^{0\nu}(^{130}\text{Te})$ is close to the current experimental lower limit, the positivity constraint intervals of values of $T_{1/2}^{0\nu}(^{136}\text{Xe})$ for the each of the three pairs of interfering mechanisms considered are incompatible with the EXO lower bound on $T_{1/2}^{0\nu}(^{136}\text{Xe})$, eq. (20).

We consider next a case in which the half-life of ^{136}Xe is one of the two half-lives assumed to have been experimentally determined. The $(\beta\beta)_{0\nu}$ -decay is supposed to be triggered by light Majorana neutrino and gluino exchange mechanisms with LFV parameters $|\eta_\nu|^2$ and $|\eta_{\chi'}|^2$. We use in the analysis the half-lives of ^{76}Ge , ^{136}Xe and ^{130}Te , which will be denoted for simplicity respectively as T_1 , T_2 and T_3 . Once the experimental bounds on T_i , $i = 1, 2, 3$, given in eq. (15), are taken into account, the conditions for destructive interference, i.e., for $\cos \alpha < 0$, are given by:

$$z < 0 : \begin{cases} 1.9 \times 10^{25} < T_1 \leq 1.90T_2, & T_3 \geq \frac{9.64T_1T_2}{16.32T_1 + 8.59T_2}; \\ 1.90T_2 < T_1 \leq 2.78T_2, & T_3 > \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}; \\ T_1 > 2.78T_2, & T_3 \geq \frac{7.33T_1T_2}{11.94T_1 + 7.61T_2}, \end{cases} \quad (31)$$

where we have used the “large basis” $g_A = 1.25$ Argonne NMEs. The conditions for constructive interference read:

$$z > 0 : \begin{cases} 1.90T_2 < T_1 \leq 2.29T_2, & \frac{9.64T_1T_2}{16.32T_1 + 8.59T_2} \leq T_3 \leq \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}; \\ 2.29T_2 < T_1 < 2.78T_2, & \frac{7.33T_1T_2}{11.94T_1 + 7.61T_2} \leq T_3 \leq \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}. \end{cases} \quad (32)$$

TABLE III. Ranges of the half-life of ^{136}Xe for different fixed values of the half-lives of ^{76}Ge and ^{130}Te in the case of three pairs of interfering mechanisms: light Majorana neutrino exchange and gluino exchange (upper table); light Majorana and heavy LH Majorana neutrino exchanges (lower table). The results shown are obtained with the “large basis” $g_A = 1.25$ Argonne NMEs. Two stars indicates that the EXO bound rules out the corresponding solution.

$T_{1/2}^{0\nu}[\text{y}](\text{fixed})$	$T_{1/2}^{0\nu}[\text{y}](\text{fixed})$	Allowed Range
$T(\text{Ge}) = 2.23 \cdot 10^{25} **$	$T(\text{Te}) = 3 \cdot 10^{24}$	$2.95 \cdot 10^{24} \leq T(\text{Xe}) \leq 5.65 \cdot 10^{24}$
$T(\text{Ge}) = 10^{26} **$	$T(\text{Te}) = 3 \cdot 10^{24}$	$3.43 \cdot 10^{24} \leq T(\text{Xe}) \leq 4.66 \cdot 10^{24}$
$T(\text{Ge}) = 2.23 \cdot 10^{25}$	$T(\text{Te}) = 3 \cdot 10^{25}$	$1.74 \cdot 10^{25} \leq T(\text{Xe}) \leq 1.66 \cdot 10^{26}$
$T(\text{Ge}) = 10^{26}$	$T(\text{Te}) = 3 \cdot 10^{25}$	$2.58 \cdot 10^{25} \leq T(\text{Xe}) \leq 6.90 \cdot 10^{25}$

$T_{1/2}^{0\nu}[\text{y}](\text{fixed})$	$T_{1/2}^{0\nu}[\text{y}](\text{fixed})$	Allowed Range
$T(\text{Ge}) = 2.23 \cdot 10^{25} **$	$T(\text{Te}) = 3 \cdot 10^{24}$	$4.93 \cdot 10^{24} \leq T(\text{Xe}) \leq 6.21 \cdot 10^{24}$
$T(\text{Ge}) = 10^{26} **$	$T(\text{Te}) = 3 \cdot 10^{24}$	$5.23 \cdot 10^{24} \leq T(\text{Xe}) \leq 5.83 \cdot 10^{24}$
$T(\text{Ge}) = 2.23 \cdot 10^{25}$	$T(\text{Te}) = 3 \cdot 10^{25}$	$3.95 \cdot 10^{25} \leq T(\text{Xe}) \leq 8.25 \cdot 10^{25}$
$T(\text{Ge}) = 10^{26}$	$T(\text{Te}) = 3 \cdot 10^{25}$	$4.68 \cdot 10^{25} \leq T(\text{Xe}) \leq 6.61 \cdot 10^{25}$

If we set, e.g., the ^{76}Ge half-life to the value claimed in [48] $T_1 = 2.23 \times 10^{25}$ y, we find that only destructive interference between the contributions of the two mechanisms considered in the $(\beta\beta)_{0\nu}$ -decay rate, is possible. Numerically we get in this case

$$T_3 > \frac{3.44T_2}{5.82 + 1.37 \times 10^{-25}T_2}. \quad (33)$$

For $1.37 \times 10^{-25}T_2 \ll 5.82$ one finds:

$$T(^{130}\text{Te}) \gtrsim 0.59 T(^{136}\text{Xe}) \gtrsim 9.46 \times 10^{24} \text{ y}, \quad (34)$$

where the last inequality has been obtained using the EXO lower bound on $T(^{136}\text{Xe})$. Constructive interference is possible for the pair of interfering mechanisms under discussion only if $T(^{76}\text{Ge}) \gtrsim 3.033 \times 10^{25}$ y.

The possibilities of destructive and constructive interference are illustrated in Figs. 7 and 8, respectively. In these figures the physical allowed regions, determined through the positivity conditions, correspond to the areas within the two vertical lines (the solutions must be compatible also with the existing lower limits given in eq (15)). For instance, using the Argonne “large basis” NMEs corresponding to $g_A = 1.25$ and setting $T(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y and $T(^{130}\text{Te}) = 10^{25}$ y, positive solutions are allowed only in the interval $1.60 \times 10^{25} \leq T(^{136}\text{Xe}) \leq 2.66 \times 10^{25}$ y (Fig. 7). As can be seen in Figs. 7 and 8, a constructive interference is possible only if $T_2 \equiv T(^{136}\text{Xe})$ lies in a relatively narrow interval and $T_3 \equiv T(^{130}\text{Te})$ is determined through the conditions in eq. (32).

Next, we would like to illustrate the possibility to distinguish between two pairs of interfering mechanisms i) A+B and ii) B+C, which share one mechanism, namely B, from the data on the half-lives of three isotopes. In this case we can set two systems of three equations, each one in three unknowns. We will denote the corresponding LNV parameters as i) $|\eta_A|^2$, $|\eta_B|^2$ and ii) $|\eta_B|^2$ and $|\eta_C|^2$, while the interference parameters will be denoted as i) z and ii) z' . Fixing two of the three half-lives, say T_i and T_j , the possibility to discriminate between the mechanisms A and C relies on the dependence of $|\eta_A|^2$ and $|\eta_C|^2$ on the third half-life, T_k . Given T_i and T_j , it will be possible to discriminate between the mechanisms A and C if the two intervals of values of T_k where $|\eta_A|^2 > 0$ and $|\eta_C|^2 > 0$, do not overlap. If, instead, the two intervals partially overlap, complete discrimination would be impossible, but there would be a large interval of values of T_k (or equivalently, positive solutions values of the LNV parameters) that can be excluded using present or future experimental data. In order to have non-overlapping positive solution intervals of T_K , corresponding to $|\eta_A|^2 > 0$

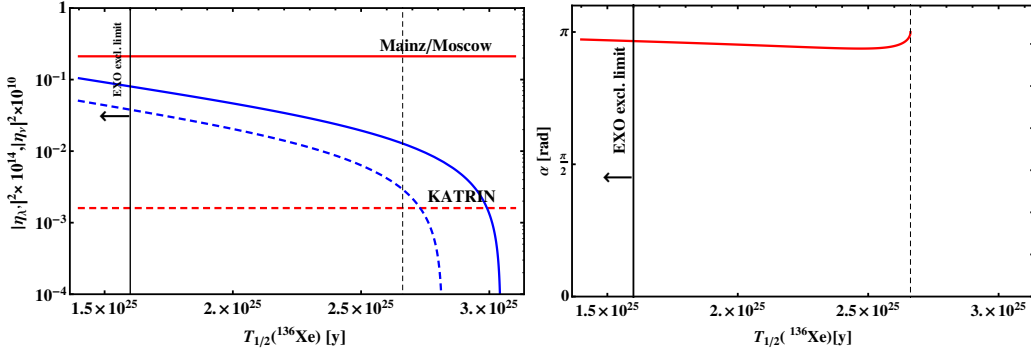


FIG. 7. Left panel: the values of $|\eta_\nu|^2 \times 10^{10}$ (thick solid line) and $|\eta_{\chi'}|^2 \times 10^{14}$ (dotted line), obtained as solutions of the system of equations (14) for fixed values of $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y and $T_{1/2}^{0\nu}(^{130}\text{Te}) = 10^{25}$ y, and letting $T_{1/2}^{0\nu}(^{136}\text{Xe})$ free. The physical allowed regions correspond to the areas within the two vertical lines. Right panel: the values of the phase α in the allowed interval of values of $T_{1/2}^{0\nu}(^{136}\text{Xe})$, corresponding to physical solutions for $|\eta_\nu|^2$ and $|\eta_{\chi'}|^2$. In this case $\cos \alpha < 0$ and the interference is destructive. See text for details.

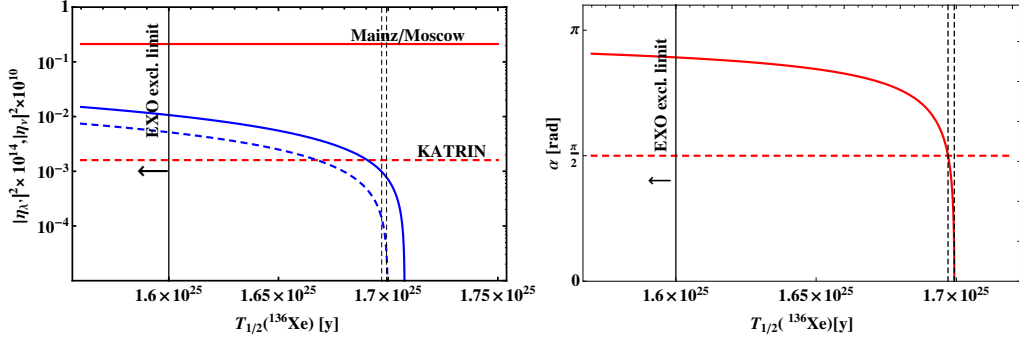


FIG. 8. Left panel: the same as in Fig. 7 but for $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 3.5 \times 10^{25}$ y and $T_{1/2}^{0\nu}(^{130}\text{Te}) = 8.0 \times 10^{25}$ y. The interval of values of $T_{1/2}^{0\nu}(^{136}\text{Xe})$ between i) the vertical solid and right dashed lines ii) the two vertical dashed lines, and iii) the vertical solid and left dashed lines, correspond respectively to i) physical (non-negative) solutions for $|\eta_\nu|^2$ and $|\eta_{\chi'}|^2$, ii) constructive interference ($z > 0$), and iii) destructive interference ($z < 0$). Right panel: the corresponding values of the phase α as a function of $T_{1/2}^{0\nu}(^{136}\text{Xe})$. Constructive interference is possible only for values of $T_{1/2}^{0\nu}(^{136}\text{Xe})$ between the two vertical dashed lines. See text for details.

and $|\eta_C|^2 > 0$, the following inequality must hold:

$$\frac{(\mathcal{M}_{k,A}^{0\nu}\mathcal{M}_{i,B}^{0\nu} - \mathcal{M}_{i,A}^{0\nu}\mathcal{M}_{k,B}^{0\nu})(\mathcal{M}_{k,A}^{0\nu}\mathcal{M}_{j,B}^{0\nu} - \mathcal{M}_{j,A}^{0\nu}\mathcal{M}_{k,B}^{0\nu})}{(\mathcal{M}_{k,B}^{0\nu}\mathcal{M}_{i,C}^{0\nu} - \mathcal{M}_{i,B}^{0\nu}\mathcal{M}_{k,C}^{0\nu})(\mathcal{M}_{k,B}^{0\nu}\mathcal{M}_{j,C}^{0\nu} - \mathcal{M}_{j,B}^{0\nu}\mathcal{M}_{k,C}^{0\nu})} < 0. \quad (35)$$

The above condition can be satisfied only for certain sets of isotopes. Obviously, whether it is fulfilled or not depends on the values of the relevant NMEs. We will illustrate this on the example of an over-simplified analysis involving the light Majorana neutrino exchange, the heavy LH Majorana neutrino exchange and the gluino exchange as mechanisms A , B and C , respectively, and the half-lives of ^{76}Ge , ^{130}Te and ^{136}Xe : $T_1 \equiv T(^{76}\text{Ge})$, $T_2 \equiv T(^{130}\text{Te})$ and $T_3 \equiv T(^{136}\text{Xe})$. Fixing $T_1 = 2.23 \times 10^{25}$ y and $T_3 = 1.6 \times 10^{25}$ y (the EXO 90% C.L. lower limit), we obtain the results shown in Fig. 9. As it follows from Fig. 9, in the case of the Argonne NMEs (left panel), it is possible to discriminate between the standard light neutrino exchange and the gluino exchange mechanisms: the intervals of values of T_2 , where the positive solutions for the LNV parameters of the two pairs of interfering mechanisms considered occur, do not overlap. Further, the physical solutions for the two LNV parameters related to the gluino mechanism are excluded by the CUORICINO limit on $T(^{130}\text{Te})$ [47]. This result does not change with the increasing of T_3 . Thus, we are lead to conclude that for $T_3 > 1.6 \times 10^{25}$ y and

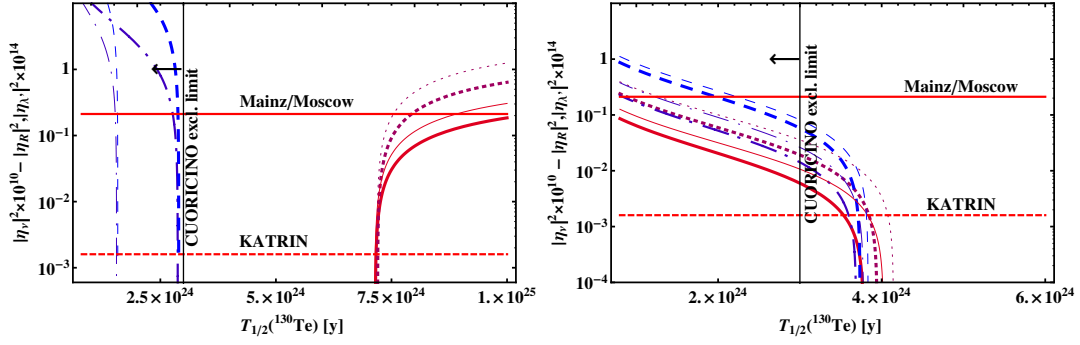


FIG. 9. The parameters $|\eta_\nu|^2 \times 10^{10}$ (solid line) and $|\eta_L|^2 \times 10^{14}$ (dotted line) of the light and heavy LH Majorana neutrino exchange mechanisms, and $|\eta_{\lambda'}|^2 \times 10^{14}$ (dashed-dotted line) and $|\eta_L|^2 \times 10^{14}$ (dashed line) of the gluino and heavy LH Majorana neutrino exchange mechanisms, obtained from eq. (27) using the Argonne NMEs (left panel) and CD-Bonn NMEs (right panel), corresponding to $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines), for $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}\text{y}$, $T_{1/2}^{0\nu}(^{136}\text{Xe}) = 1.60 \times 10^{25}\text{y}$ and letting $T_{1/2}^{0\nu}(^{130}\text{Te})$ free. See text for details.

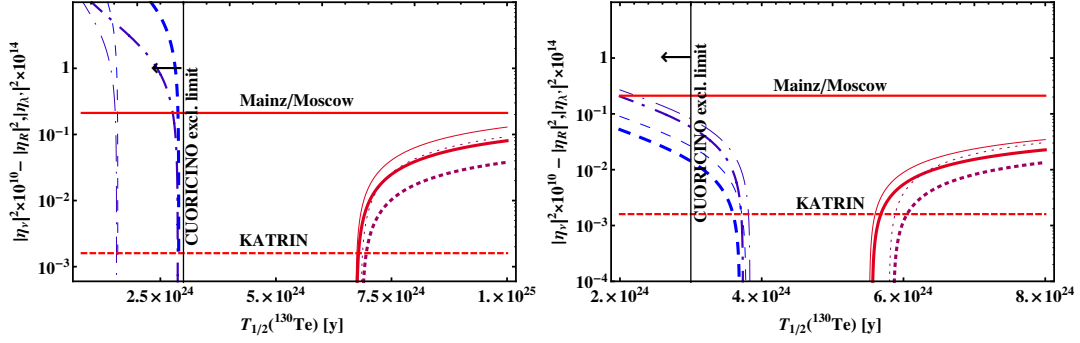


FIG. 10. The same as in Fig. 9, but for i) $|\eta_\nu|^2 \times 10^{10}$ (thick solid line) and $|\eta_{\lambda'}|^2 \times 10^{14}$ (thick dotted line) of the light neutrino and gluino exchange mechanisms, and ii) $|\eta_L|^2 \times 10^{14}$ (thick dashed-dotted line) and $|\eta_{\lambda'}|^2 \times 10^{14}$ (thick dashed line) of the heavy LH Majorana neutrino and gluino exchange mechanisms, and using $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}\text{y}$ and $T_{1/2}^{0\nu}(^{136}\text{Xe}) = 1.60 \times 10^{25}\text{y}$. See text for details.

T_1 given by the value claimed in [48], of the two considered pairs of possible interfering $(\beta\beta)_{0\nu}$ -decay mechanisms, only the light and heavy LH Majorana neutrino exchanges can be generating the decay. The solution for $|\eta_\nu|^2$ must be compatible with the upper limit $|\langle m \rangle| < 2.3\text{ eV}$ [16, 50], indicated with a solid horizontal line in Fig. 9. In the right panel of Fig. 9 we plot also the solutions obtained with the CD-Bonn NMEs. In this case is not possible to discriminate between the two considered pair of mechanisms since the condition in eq. (35) is not satisfied.

Another interesting example is the case in which A is the light Majorana neutrino exchange, B is the gluino exchange and C the heavy LH Majorana neutrino exchange, i.e., we try to discriminate between i) the light neutrino plus gluino exchange mechanisms, and ii) the heavy LH Majorana neutrino plus gluino exchange mechanisms. We fix, like in the previous case, the values for $T_1 = 2.23 \times 10^{25}\text{y}$ and $T_3 = 1.6 \times 10^{25}\text{y}$. The results of this analysis are plotted in Fig. 10. Since the condition in eq. (35) is now satisfied for NMEs obtained either with the Argonne potential or with the CD-Bonn potential, in this case it is possible, in principle, to discriminate between the two pair of mechanisms independently of the set of NMEs used (within the sets considered by us). This result does not change with the increasing of T_3 . Hence, as far as T_1 is fixed to the value claimed in [48] and the limits in eq. (15) are satisfied, the two intervals of values of T_2 , in which the “positivity conditions” for i) $|\eta_\nu|^2$, $|\eta_{\lambda'}|^2$ and z , and for ii) $|\eta_{\lambda'}|^2$, $|\eta_N|^2$ and z' , are satisfied, are not overlapping (Fig. 10).

VIII. FINAL REMARKS

We have investigated the possibility to discriminate between different pairs of CP non-conserving mechanisms inducing the neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay by using a multi-isotope approach. In particular, data on $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different NMEs can be used to obtain information on couple of (non)-interfering mechanisms in $(\beta\beta)_{0\nu}$ -decay. The mechanisms studied are: light Majorana neutrino exchange, heavy left-handed (LH) and heavy right-handed (RH) Majorana neutrino exchanges, lepton charge non-conserving couplings in SUSY theories with R -parity breaking giving rise to the “dominant gluino exchange”. Each of the mechanisms is characterized by a specific lepton number violating (LNV) parameter η_κ , where the index κ labels the mechanism. For the five mechanisms listed above we use the notations $\kappa = \nu, L, R, \lambda'$, respectively. The parameter η_κ will be complex, in general, if the mechanism κ does not conserve the CP symmetry. The nuclei considered are ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe .

Four sets of NMEs of the $(\beta\beta)_{0\nu}$ -decays of these five nuclei, derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA), were employed in our analysis. They correspond to two types of nucleon-nucleon potentials - Argonne (“Argonne NMEs”) and CD-Bonn (“CD-Bonn NMEs”), and two values of the axial coupling constant $g_A = 1.25; 1.00$. Given the NMEs and the phase space factors of the decays, the half-life of a given nucleus depends on the parameters $|\eta_\kappa|^2$ of the mechanisms triggering the decay.

We have considered in detail the cases of two non-interfering and two interfering mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay. If two non-interfering mechanisms A and B cause the decay, the parameters $|\eta_A|^2$ and $|\eta_B|^2$ can be determined from data on the half-lives of two isotopes, T_1 and T_2 as solutions of a system of two linear equations. If the half-life of one isotope is known, say T_1 , the positivity condition which the solutions $|\eta_A|^2$ and $|\eta_B|^2$ must satisfy, $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$, constrain the half-life of the second isotope T_2 (and the half-life of any other isotope for that matter) to lie in a specific interval. If A and B are interfering mechanisms, $|\eta_A|^2$ and $|\eta_B|^2$ and the interference term parameter, $z_{AB} \equiv 2 \cos \alpha_{AB} |\eta_A \eta_B|$ which involves the cosine of an unknown relative phase α_{AB} of η_A and η_B , can be uniquely determined, in principle, from data on the half-lives of three nuclei, $T_{1,2,3}$. In this case, given the half-life of one isotope, say T_1 , the “positivity conditions” $|\eta_A|^2 \geq 0$, $|\eta_B|^2 \geq 0$ and $-1 \leq \cos \alpha_{AB} \leq 1$ constrain the half-life of a second isotope, say T_2 , to lie in a specific interval, and the half-life of a third one, T_3 , to lie in an interval which is determined by the value of T_1 and the interval of allowed values of T_2 .

For all possible pairs of non-interfering mechanisms we have considered these “positivity condition” intervals of values of T_2 were shown to be essentially degenerate if T_1 and T_2 correspond to the half-lives of any pair of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . This is a consequence of the fact that for each of the five single mechanisms discussed, the NMEs for ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te differ relatively no more than 10% [31, 37]. One has similar degeneracy of “positivity condition” intervals T_2 and T_3 in the cases of two constructively interfering mechanisms (within the set considered). These degeneracies might irreparably plague the interpretation of the $(\beta\beta)_{0\nu}$ -decay data if the process will be observed.

The NMEs for ^{136}Xe , resulting from calculations which use the SRQRPA method, differ significantly from those of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , being by a factor $\sim (1.3 - 2.5)$ smaller. As we have shown in the present section, this allows to lift to a certain degree the indicated degeneracies and to draw conclusions about the pair of non-interfering (interfering) mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and of at least one (two) more isotope(s) which can be, e.g., any of the four, ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te considered.

We have analyzed also the possibility to discriminate between two pairs of non-interfering (or interfering) $(\beta\beta)_{0\nu}$ -decay mechanisms when the pairs have one mechanism in common, i.e., between the mechanisms i) $A + B$ and ii) $C + B$, using the half-lives of the same two isotopes. We have derived the general conditions under which it would be possible, in principle, to identify which pair of mechanisms is inducing the decay (if any). We have shown that the conditions of interest are fulfilled, e.g., for the following two pairs of non-interfering mechanisms i) light neutrino exchange (A) and heavy

RH Majorana neutrino exchange (B) and ii) gluino exchange (C) and heavy RH Majorana neutrino exchange (B), and for the following two pairs of interfering mechanisms i) light neutrino exchange (A) and heavy LH Majorana neutrino exchange (B) and ii) gluino exchange (C) and heavy LH Majorana neutrino exchange (B), if one uses the Argonne NMEs in the analysis. They are fulfilled for both the Argonne NMEs and CD-Bonn NMEs, e.g., for the following two pairs of interfering mechanisms i) light neutrino exchange (A) and gluino exchange (B), and ii) heavy LH Majorana neutrino exchange (C) and gluino exchange (B).

The results obtained here show that using the $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different NMEs would help resolving the problem of identifying the mechanisms triggering the decay.

Concluding, an eventual observation of a Majorana field would be a fundamental headway. This would mean that Nature admits the existence of particles which are identical to their anti-particles and, more importantly, it could point to the existence of New Physics, or in other words to new lepton number violating couplings in the Lagrangian of particle interactions. The data on $(\beta\beta)_{0\nu}$ -decay, which will be available from the currently running experiments GERDA, EXO-200, KamLAND-Zen and from the CUORE experiment will be of crucial importance to identify the mechanism(s) triggering the decay if the latter will be observed. This will help to identify the New Physics beyond that predicted by the Standard Model associated with lepton charge non-conservation and the $(\beta\beta)_{0\nu}$ -decay.

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